X-Rays

Tube
Spectrum
Interaction
Detection

Introduction

- Aim of medical imaging:
  - Creating images of internal structures (and functions) in a living (human) organism
  - Preferably non (or minimally) destructive / noninvasive
  - Diagnostic implications

- Achieved by using the capability of various forms of energy to penetrate and interact with matter.

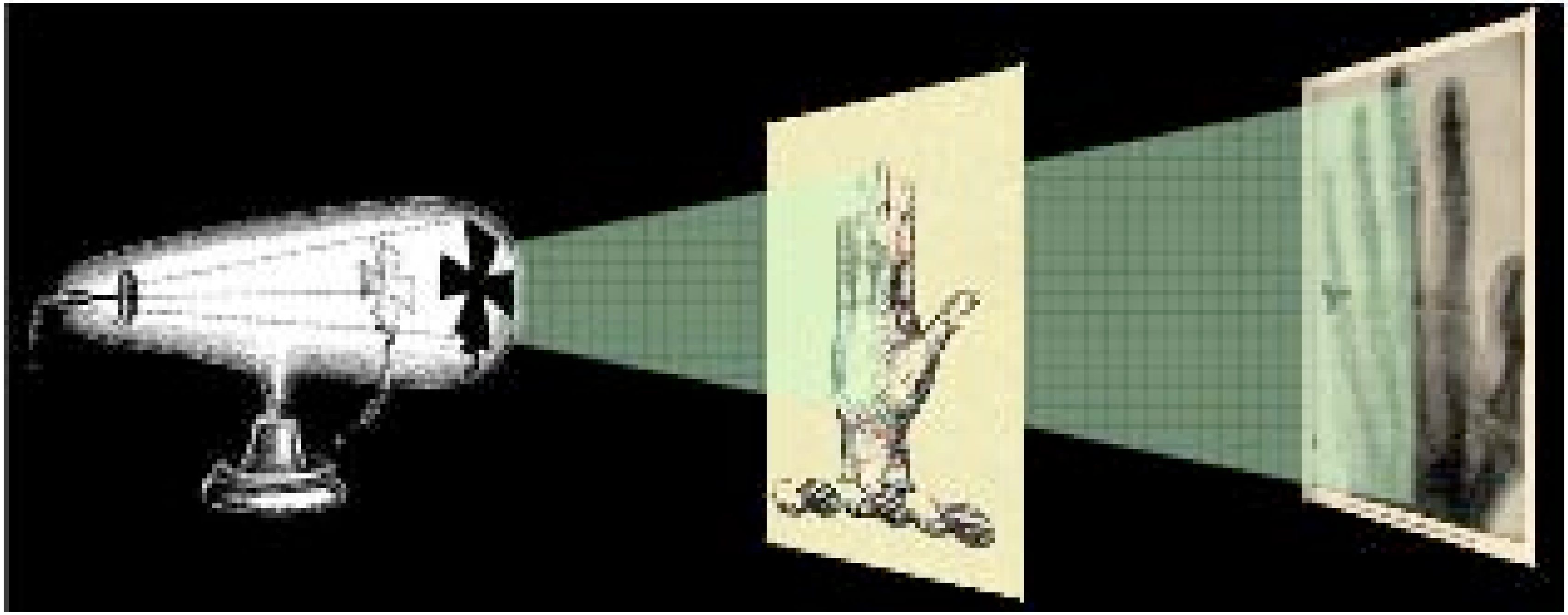
- Historic starting point:
  W. K. Röntgen's discovery of the X rays experimenting with a cathode ray tube and fluorescent screen/photographic film (1895, 1901 first Nobel award in physics).

First X-Ray Images

The famous radiograph made by Roentgen on 22 December 1895, and sent to physicist Franz Exner in Vienna. This is traditionally known as "the first X-ray picture" and "the radiograph of Mrs. Roentgen's hand. "

Radiograph of the hand of Albert von Kolliker, made at the conclusion of Roentgen's lecture and demonstration at the Würzburg Physical-Medical Society on 23 January 1896.

X-Ray Principle



X-ray Source:

- How to generate X-rays
- Parameters / optimization

Target:

- What causes contrast
- What can be imaged

Detector:

- Mechanisms
- Resolution (*x*, *t*)
- Contrast
- Sensitivity

X-rays:

- What are x-rays, • properties/description

Structure of Atoms

- Atoms composed of elementary particles
  - Nucleus: protons *p*, neutrons *n*
  - Shell: electrons *e*

- Properties of some elementary particles:

Symbol	Name	Charge [e]*	Rest Mass [amu]**
e^-, β^-	Electron	-1	0.0005486
e^+, β^+	Positron	1	0.0005486
γ	Photon	0	0
n	Neutron	0	1.0
p	Proton	1	1.0

*Elementary charge e = 1.6×10^{-19} C

**Atomic mass unit defined by ^{12}C = 12.0 amu; 1 amu = 1.66×10^{-24} kg.

Boh’r Postulate

- Classical problems with planet-like model:
  - Electron is accelerated charge ⇒ should loose energy through radiation, slow it down, and eventually crash into nucleus (atoms stable)
  - Continuous absorption and emission spectra should be allowed (discrete spectra are observed)

- Postulate:
  - Only discrete values of r allowed, corresponding to quantized angular momentum of electron $pr = l = n \times h / 2\pi$

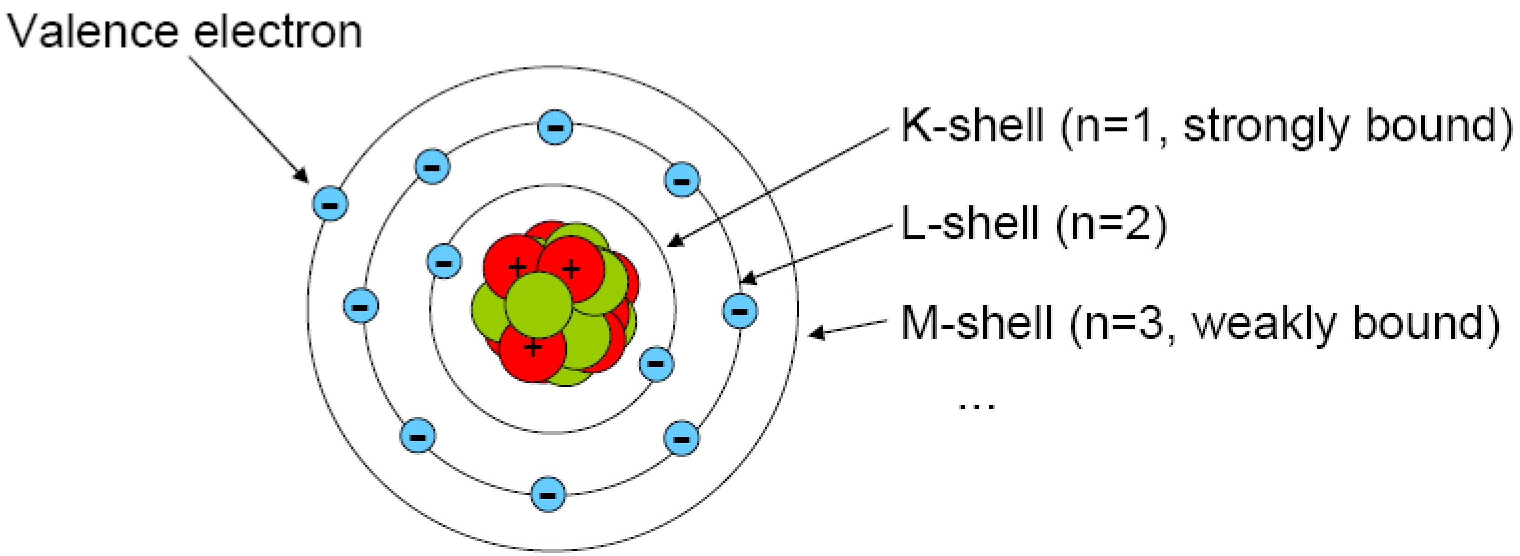
    n: quantum number, $n = 1, 2, 3, \ldots$

    h: Planck's constant, $h = 4.14 \times 10\text{-}15$ eVs
  - Transitions between orbits for absorption/emission of radiation of energy $E2 - E1 = h$ ν(resonance condition)

Complex Atoms

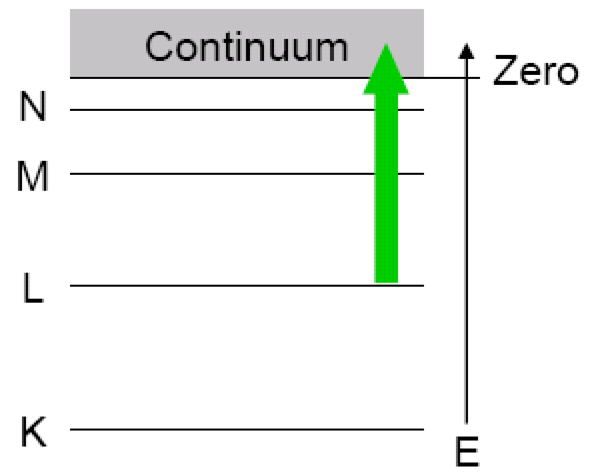
- Number of protons Z: Atomic number (determines element)
- Number of neutrons N: Neutron number
- Number of protons + neutrons $Am = Z + N$: Mass number

$$^{22}_{11}\mathrm{Na}$$



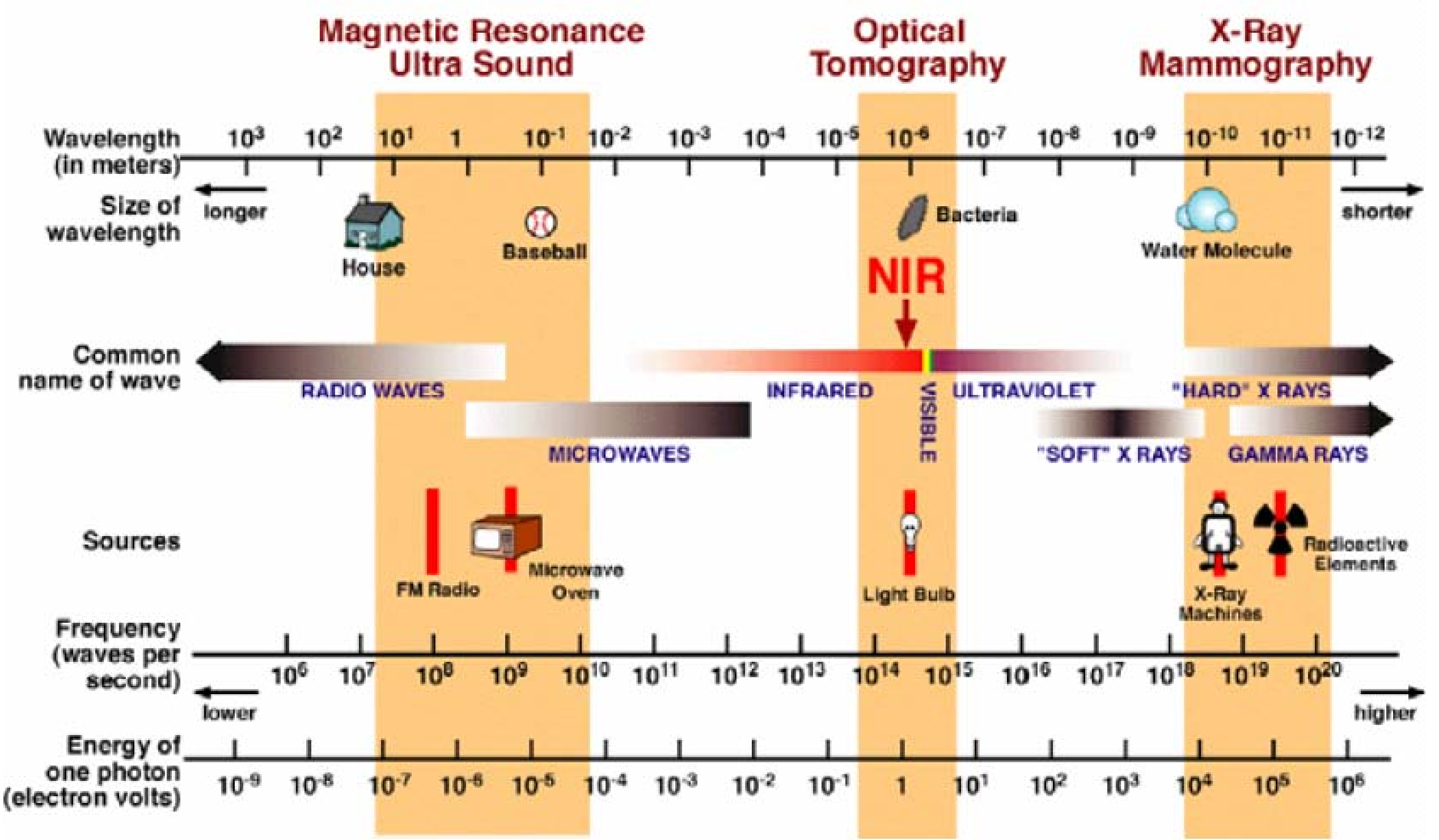
Energy Scheme

- Binding energy (BE): energy binding electron to atom
- Ionization energy $I_{K,L,...} = -BE$: amount of energy needed to remove electron from atom
- BE counted in negative units of electron volts (eV)
- At infinity, $BE = 0$.



- Binding energy for $_{53}$I: -33.2 keV (K), -4.3 keV (L), -0.6 keV (M)
- BE for valence electrons: ~ -10 eV (H: -13.6 eV)

Electromagnetic spectrum

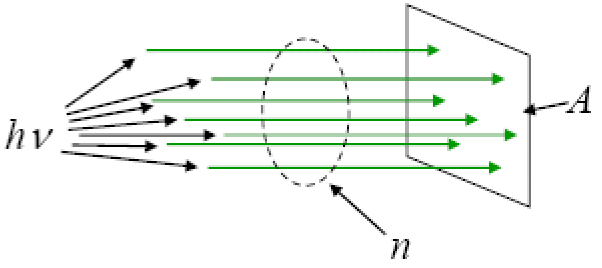


Particle Interpretation

- X-ray as electromagnetic wave:
  - Frequency $f = 1/T$ ($\omega = 2\pi f = 2\pi/T$) [s-1 = Hz]
  - Wavelength $\lambda = 2\pi/k$ [m]
  - Propagation at speed of light $c = \lambda\, v \Rightarrow$ inverse relationship of λ and v

- X-ray as particle (photon):
  - Stream of weightless, neutral particles
  - moving at speed of light
  - Each photon carries energy:

$$E_p = h\nu = hc/\lambda$$

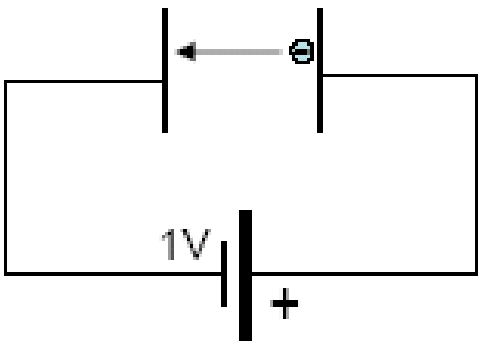


  - Radiation intensity I:

$$\frac{\text{Power}}{\text{Area}}\left[\frac{\text{W}}{\text{cm}^2}\right] = \frac{\text{Energy}}{\text{Time}\times\text{Area}}\left[\frac{\text{J}}{\text{cm}^2}\right] = \frac{\text{number of photons}\,(n)\times\text{photon energy}\,(E_p = h\nu)}{\text{Time}\times\text{Area}}$$

Energy Units

- SI unit: 1 Joule [J] = 1 Nm = 1 kg m^2 s^{-2}
- Electron volt [eV]: The potential energy of one elementary charge gained/lost ($e = 1.6 \times 10^{-19}$ C) when crossing a potential difference of 1V:

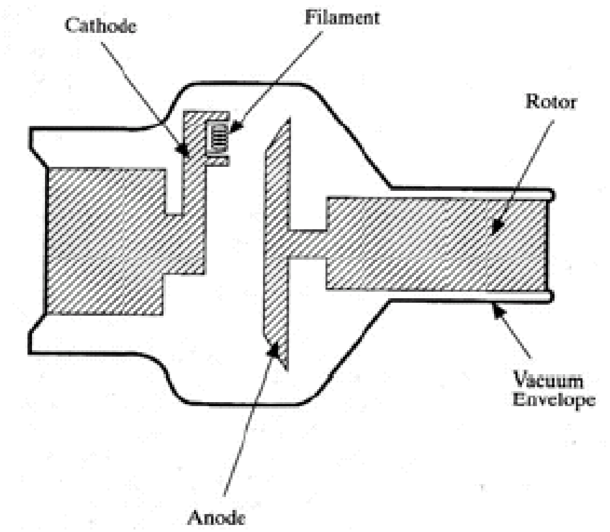


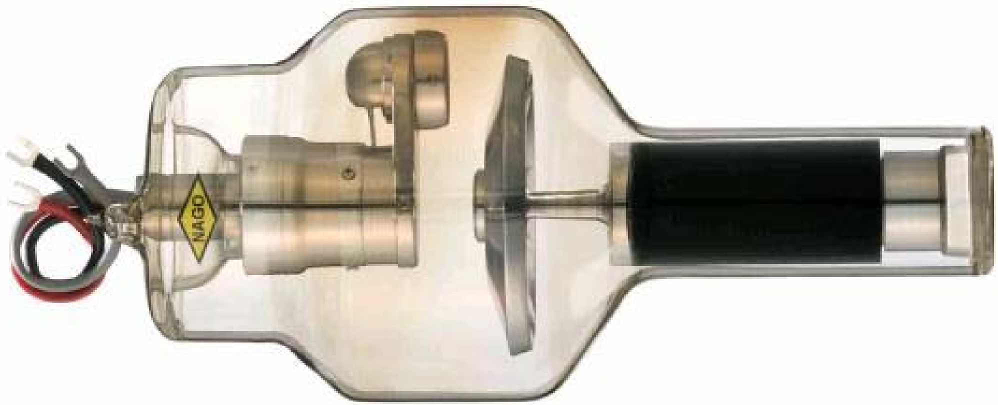
1 eV = 1.6×10-19 C ×1 V = 1.6×10-19 CV = 1.6×10-19 J

- 100 keV = 105 × 1.6×10-19 J = 1.6×10-14 J = 16 fJ (… per photon)

X-Ray tube design

- Cathode w/ focusing cup, 2 filaments (different spot sizes)
- Anode
  - Tungsten, Z_W = 74, T_{melt} = 3300 ºC
  - Embedded in copper for heat dissipation
  - Angled (see next slide)
  - Rotating to divert heat





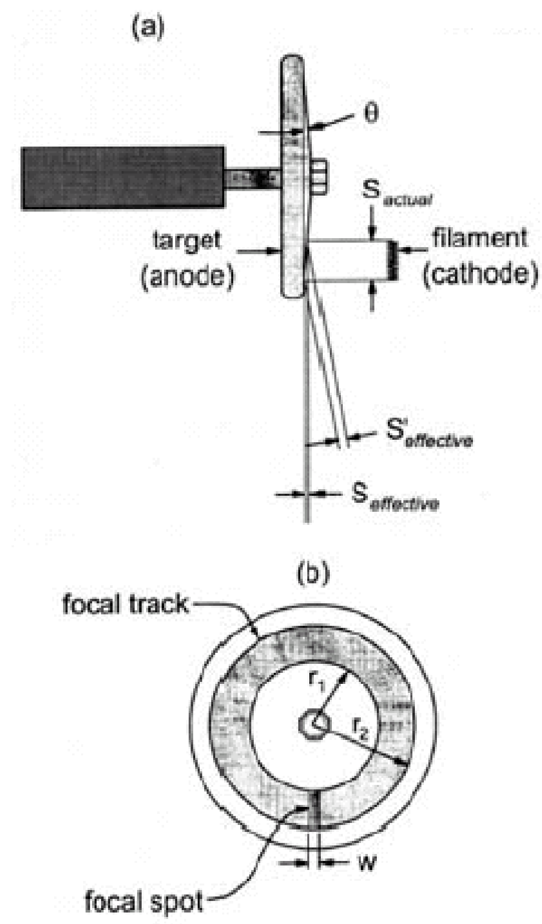
Reduction of Anode Heating

- Anode angle of 7º…15º results in apparent or effective spot size $S_{effective}$ much smaller than the actual focal spot of the electron beam (by factor ~10)

- $S_{effective}$ depending on image location

- Rotation speed ~ 1000 rpm

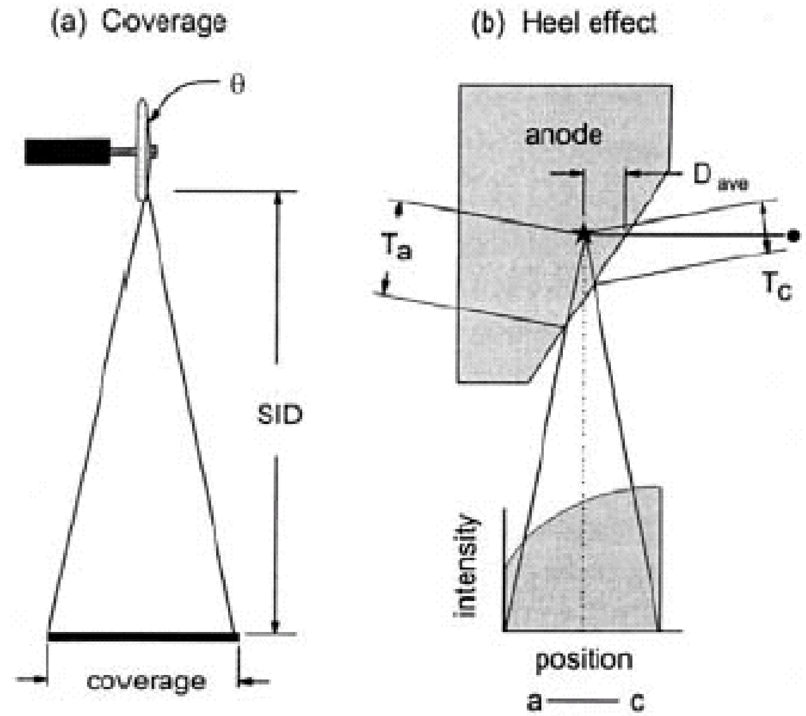
- Increases surface area for heat dissipation from $w \times (r_2 - r_1)$ to $\pi(r_2^2 - r_1^2)$; generally by a factor of 18-35.



Limits of Anode Angle

• Restricting target coverage for given source-to-image distance (SID)

• "Heel effect" causes inhomogeneous x-ray exposure



Magnification and Image Blur

Geometric magnification given by

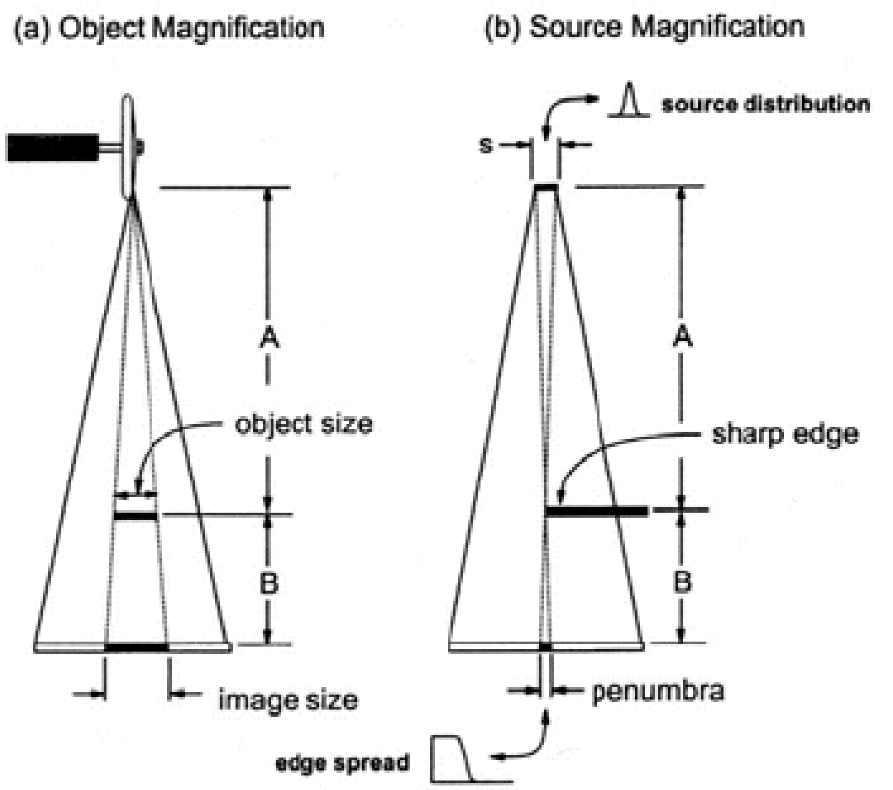
$$M = \frac{I}{O} = \frac{A+B}{A}$$

• Reduction of *Mby minimizing B, i.e. placing patient next to film. Finite target thickness can lead to variations in M.*

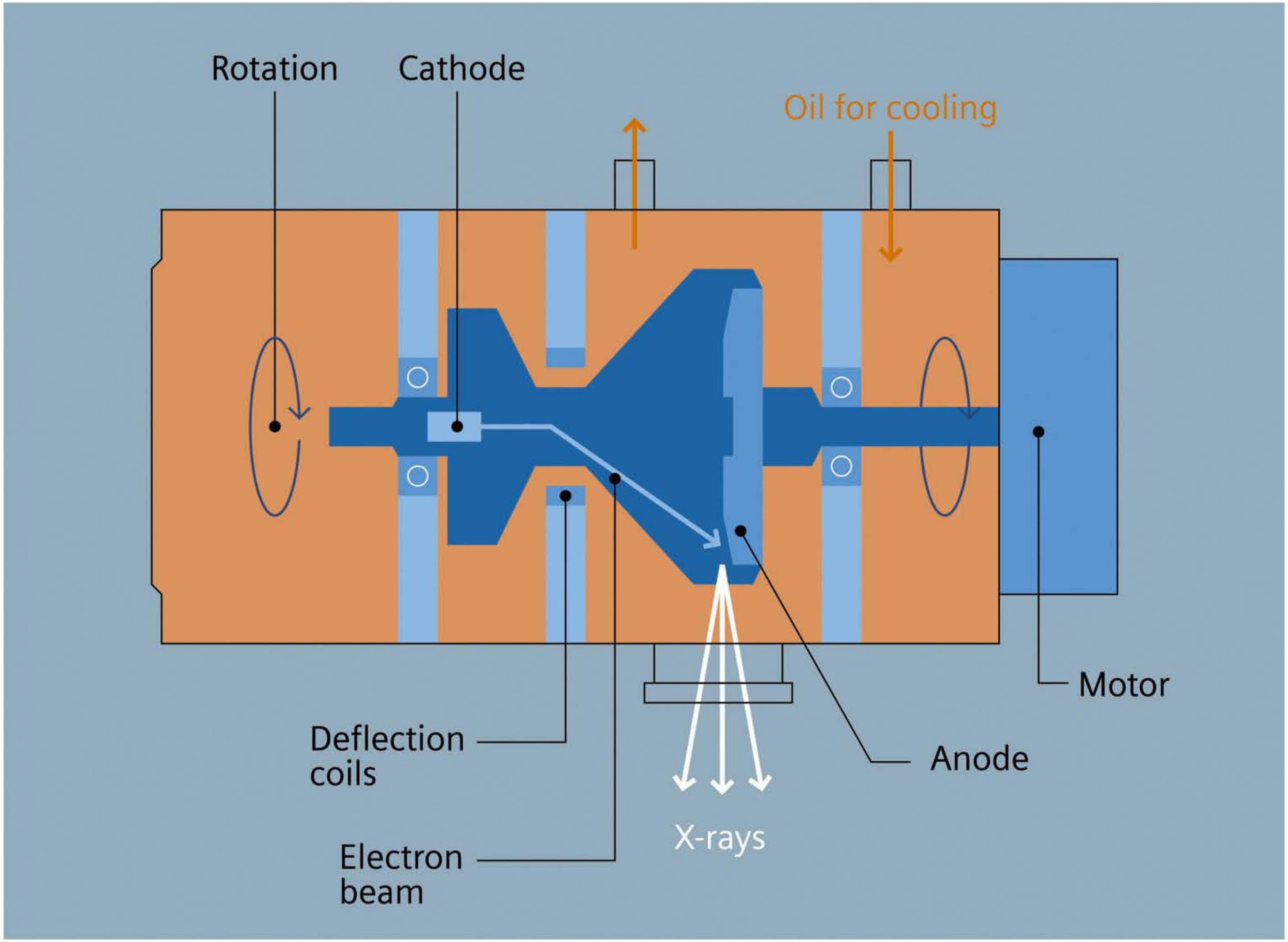
• *Blurring of edges and fine structures due to finite source size leads to penumbra p:*

$$p = s\frac{B}{A}$$

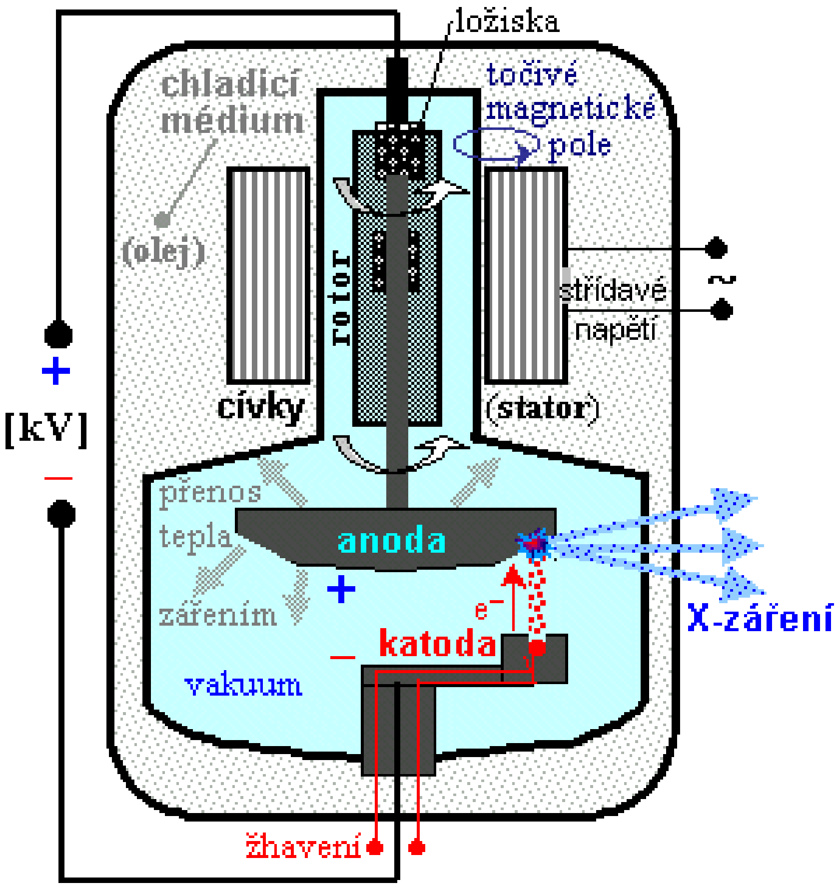
⇒loss of spatial resolution



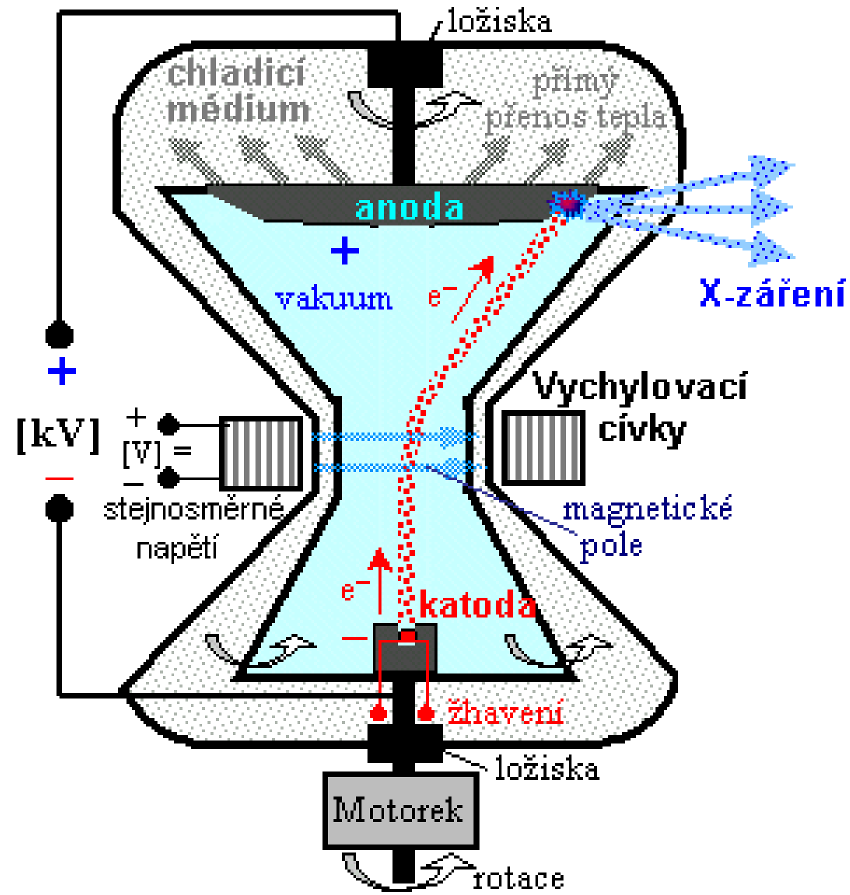
Straton X-Ray Tube



X-Ray tubes



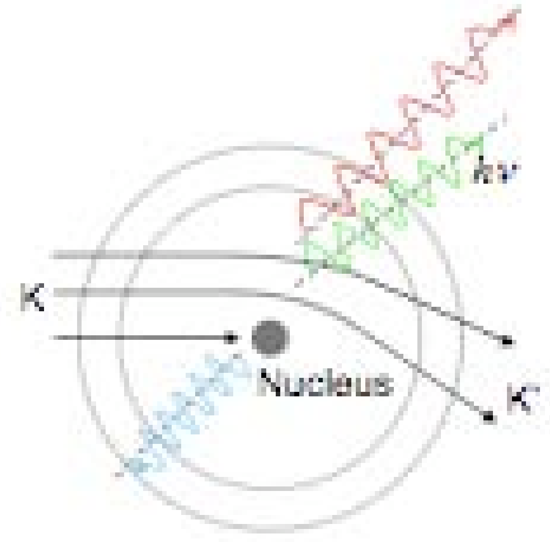
Rentgenka s excetrickou katodou a s anodou rotující uvnitř vakuové trubice



Rentgenka rotující jako celek s čelní anodou a magnetickým vychylováním elektronového svazku

Bremsstrahlung

• *Continuous* spectrum of EM radiation is produced by abrupt deceleration of charge particles ("Bremsstrahlung" is German for "braking radiation").



Deceleration is caused by deflection of electrons in the Coulomb field of the nuclei

• Most of the energy is converted into heat, ~0.5 % is x-ray

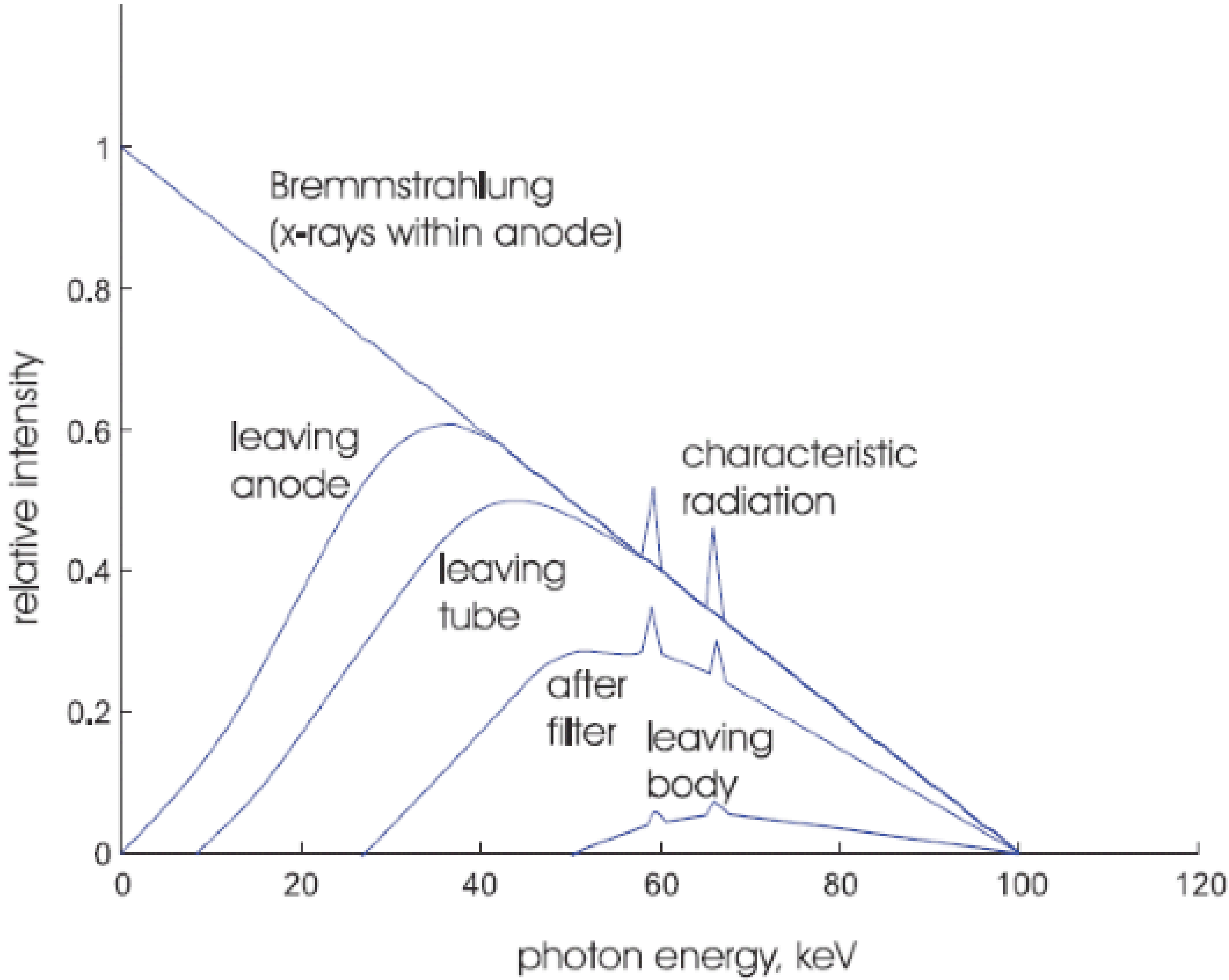
The energy of the generated x-ray photon is given by energy conservation:

$$h\nu = K_e - K'_e$$

The maximum energy for the produced photon is given by:

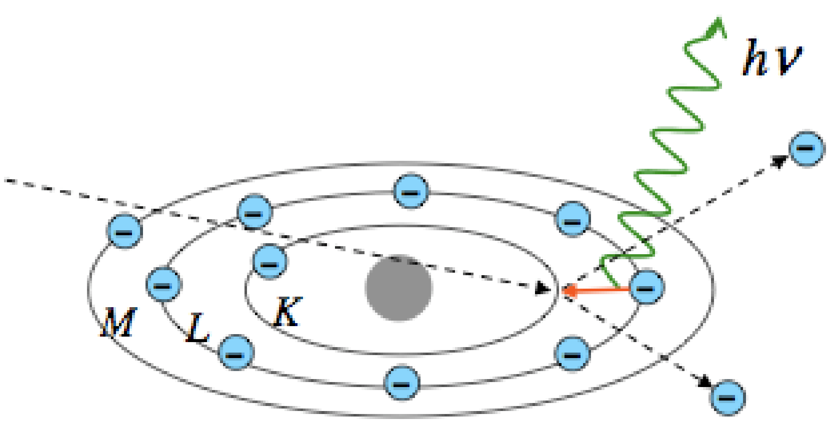
$$E_{p,\max} = h\nu = K_e = eV_{tube}$$

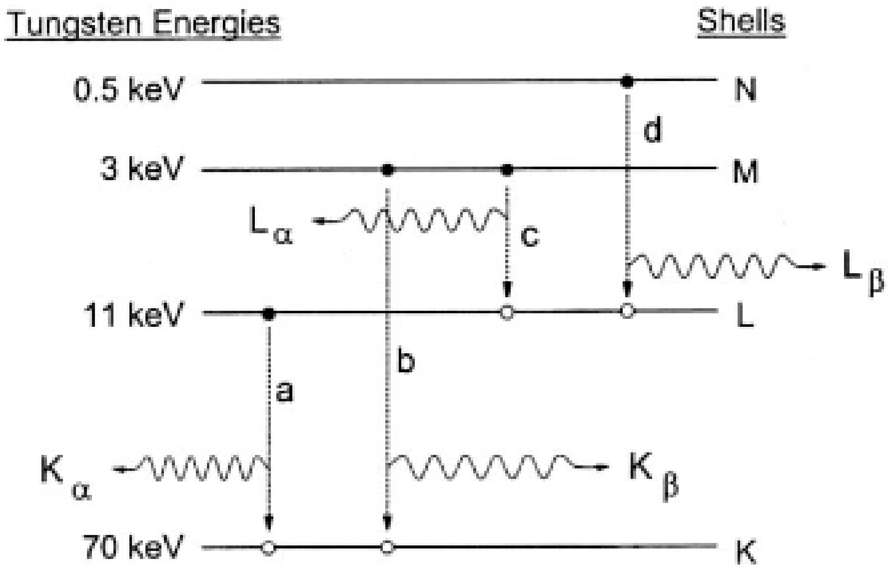
Bremsstrahlung Spectrum



Characteristic Radiation

• Narrow lines of intense x-ray at characteristic energies are superimposed on the continuous bremsstrahlung spectrum.
• Caused by removal of inner shell electrons and subsequent filling of hole with electrons from higher shell. The shell-energy difference determines the energy of characteristic rays
• Lines are named after the lower shell involved in the process; the upper shell involved is denoted by Greek letters:
$\Delta n = 1 \rightarrow \alpha$-transitions, $\Delta n = 2 \rightarrow \beta$-transitions, ...





X-Ray Spectra

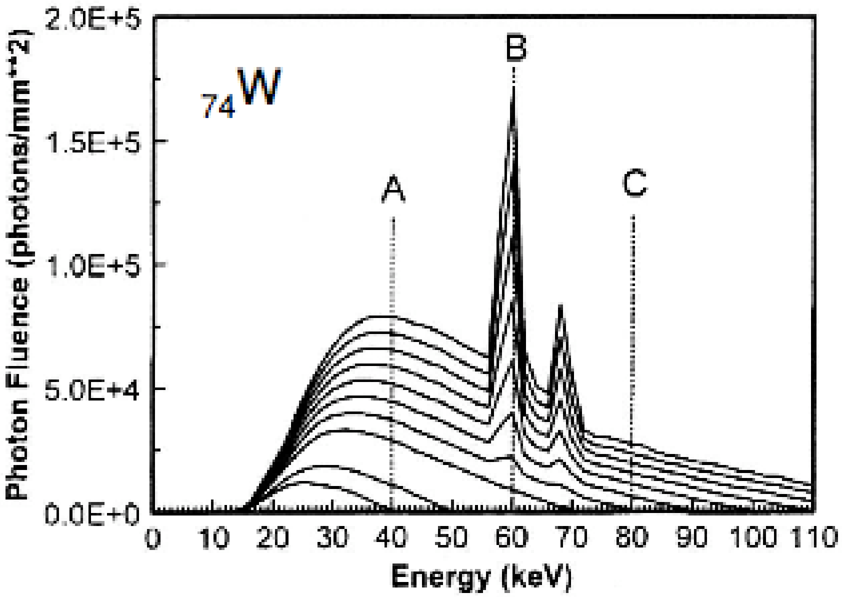
- X-ray for general diagnostic radiology produced at 40 –150 kVp

- Maximum photon energy:

$$E_p[\text{keV}] = h\nu_{\max} = e \times \text{kVp}$$

- Characteristic radiation
  occurs only for anode voltages

$$e \times \text{kVp} > I_{K,L,M,\ldots}$$



Filtration

- Low energy x-ray will be absorbed by the body, without providing diagnostic information

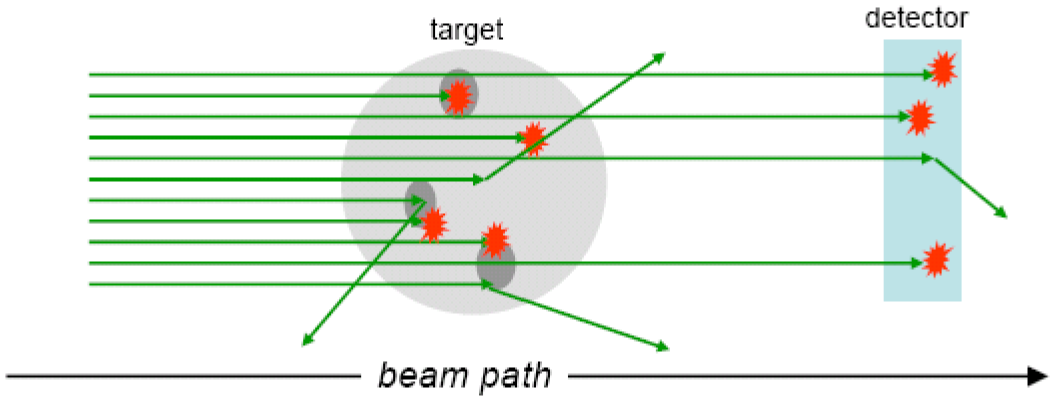
- Filtration: Process of absorbing low-energy x-ray photons before they enter the patient

- Inherent filtration
  - Within anode
  - Glass housing
- Added filtration
  - Aluminum
  - Copper/Aluminum

    Note: Cu has 8keV characteristic xrays
  - Measured in mm Al/Eq

Interaction Process

- *Absorption*: photon is destroyed and its entire energy is transferred to the target
- *Scattering*: photon is deflected and might or might not transfer portion of its energy to the target (inelastic & elastic scattering, respectively)
- Each interaction removes photon from beam path, thereby decreasing the beam intensity
- Cross-section describes interaction strength; depends on Ep and material properties (e.g., Z)



Cross-sections

- The total cross-section σ_t describes the interaction strength of radiation with matter

- Generally, σ_t is a function of
  - the physical properties of the material
  - the radiation energy

- The total cross-section is the sum of the cross-sections for different interaction (= absorption or scattering) processes:

$$\sigma_t = \sigma_1 + \sigma_2 + \sigma_3 + \dots$$

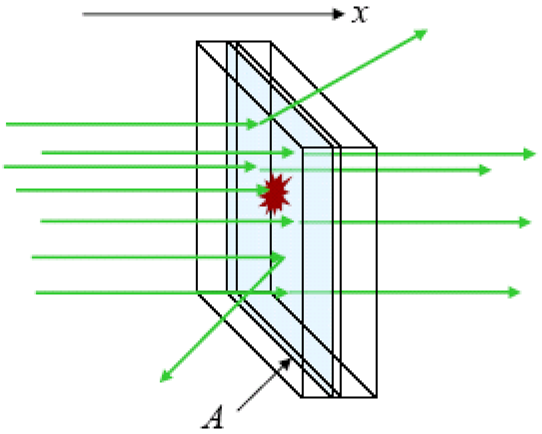
Total Cross-section I

• Consider beam of cross-sectional area *A incident on block of material*

•*The number of interactions N in a thin layer of material is proportional to:*

- *The number of atoms n covered by the beam in that layer*
- *The intensity I= n_p/A of the beam (n_p= no. of photons moving in x- direction)*

$$N \propto \frac{n}{A} n_p \quad \Rightarrow \quad N = \sigma_t \frac{n}{A} n_p = \sigma_t I \, n$$



• Proportionality constant σ_t *is called the total cross-section(unit of an area):*

$$I\ [\text{cm}^{-2}] \Rightarrow \sigma_t\ [\text{cm}^2]$$

Model assumptions:

- •Small change of intensity in the layer
- •Atoms don't "shadow" each other in layer

Total Cross-section II

The total cross-section can be interpreted as a circle of area σ_t that is centered around each atom and which has the property that a photon entering that circle will interact with that atom (= being absorbed or scattered).

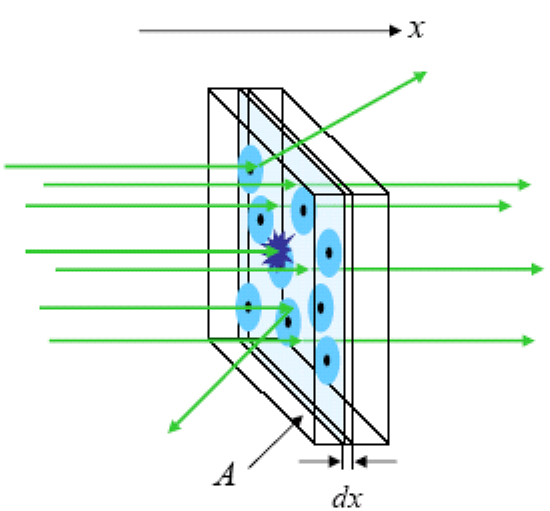
Calculate the change in beam intensity, i.e. the change in number of photons moving in *x-direction:*

The number of atoms *n in layer of thickness dx is*

$$n = \rho_n \, A \, dx \; = \; \frac{N_0}{A_m} \rho \, A \, dx$$

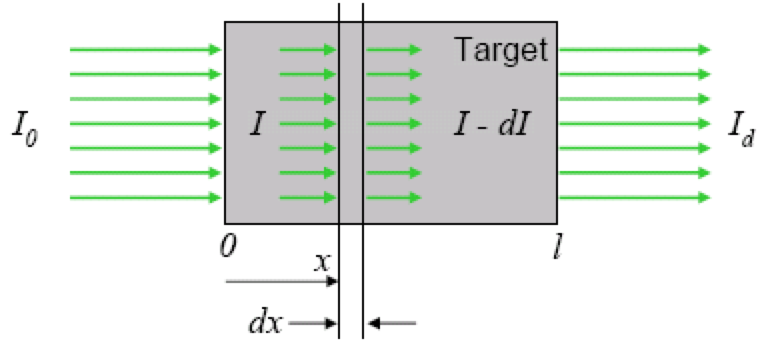
Change in intensity by interaction:

$$dI = -\frac{dn_p}{A} = -\frac{N}{A}$$



ρ_n = number density [atoms/cm^3]
ρ = density [g/cm^3]
N_0 = 6.02×10^{23} (Avogadro's constant)
A_m = atomic mass number

X-Rays in Homogeneous Target



Consider the change in intensity *dI across small distance dx:*

$$dI = -\frac{N}{A} = -\frac{\sigma_t \, n \, I}{A}$$
$$= -\sigma_t \rho_n I \, dx$$
$$= -\mu \, I \, dx$$

$$\boxed{\frac{dI}{I} = -\mu \, dx \quad \Leftrightarrow \quad \frac{dI}{dx} = -\mu \, I}$$

$\mu = \sigma_t \rho_n$*: linear attenuation coefficient*

Exponential Law of Absorption

Integration over entire target length:

$$\int_{I_0}^{I_d} \frac{dI}{I} = -\int_0^l \mu \, dx \quad \Rightarrow$$

$$\ln I_d - \ln I_0 = -\mu \, l \qquad \Leftrightarrow \quad \ln \frac{I_d}{I_0} = -\mu \, l$$

$$\Rightarrow \quad \frac{I_d}{I_0} = e^{-\mu \, l}$$

The exponential law of absorption:

$$\boxed{I(x) = I_0 e^{-\mu \, x}}$$

Linear Attenuation Coefficient

Linear attenuation coefficient μ[cm-1] depends on photon energy and material ⇒absorption spectra

Half value layer HVL = the thickness after which intensity is reduced to half of the initial value:

$$I(x = \mathrm{HVL}) = \frac{I_0}{2} \quad \Rightarrow \quad 0.5 = e^{-\mu(x=\mathrm{HVL})}$$

$$\Rightarrow \quad \ln 2 = \mu\,\mathrm{HVL}$$

$$\Leftrightarrow \quad \mathrm{HVL} = \frac{\ln 2}{\mu} = \frac{0.693}{\mu}$$

Mass-attenuation coefficient μ/ρ[cm^2/g] ⇒independent of physical state of material

Physical Interaction Process I

Three relevant interaction processes with individual cross-sections :

- Photoelectric absorption (τ)
- Compton scattering (σ)
- Pair production (κ)

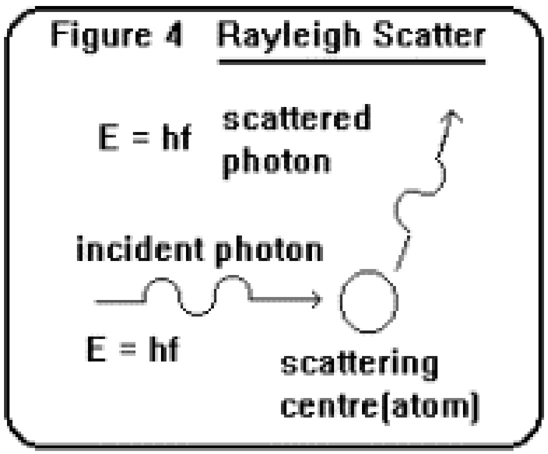
Cross sections add up:

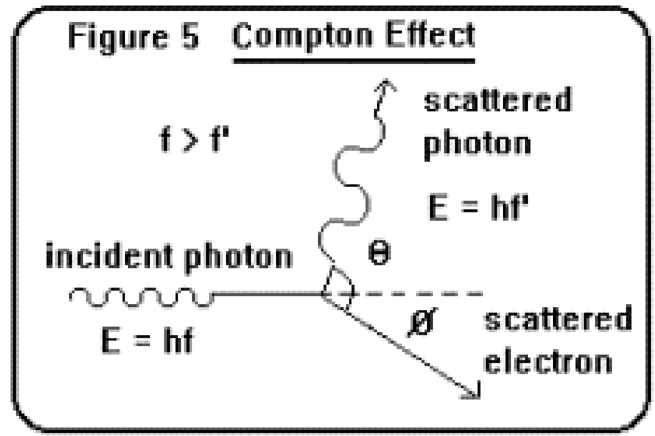
$$\sigma_t = \tau_a + \sigma_a + \kappa_a$$

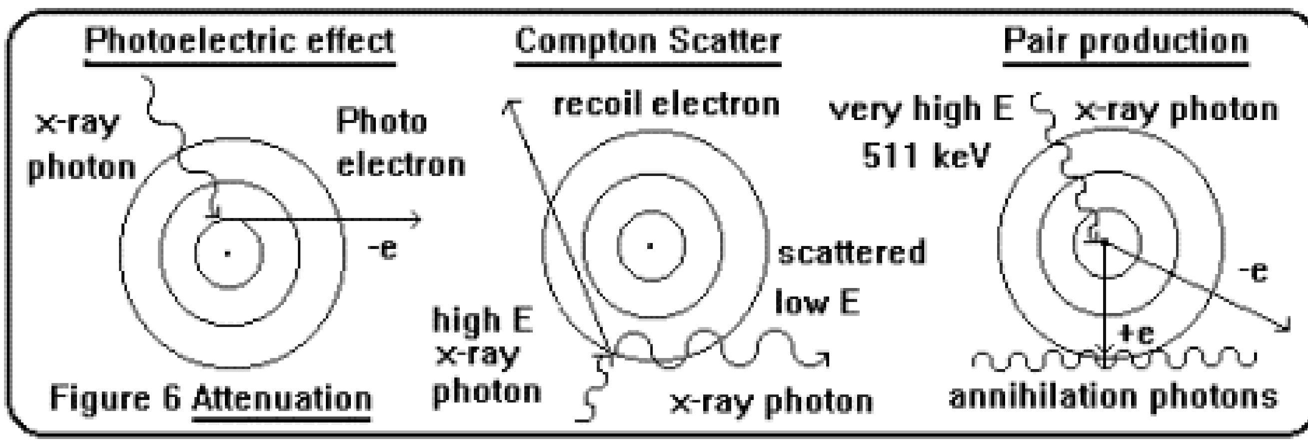
⇒linear absorption coefficient is the sum of individual absorption and scattering coefficients

$$\mu = (\tau_a + \sigma_a + \kappa_a)\,\rho_n = \tau + \sigma + \kappa$$

Physical Interaction Process II

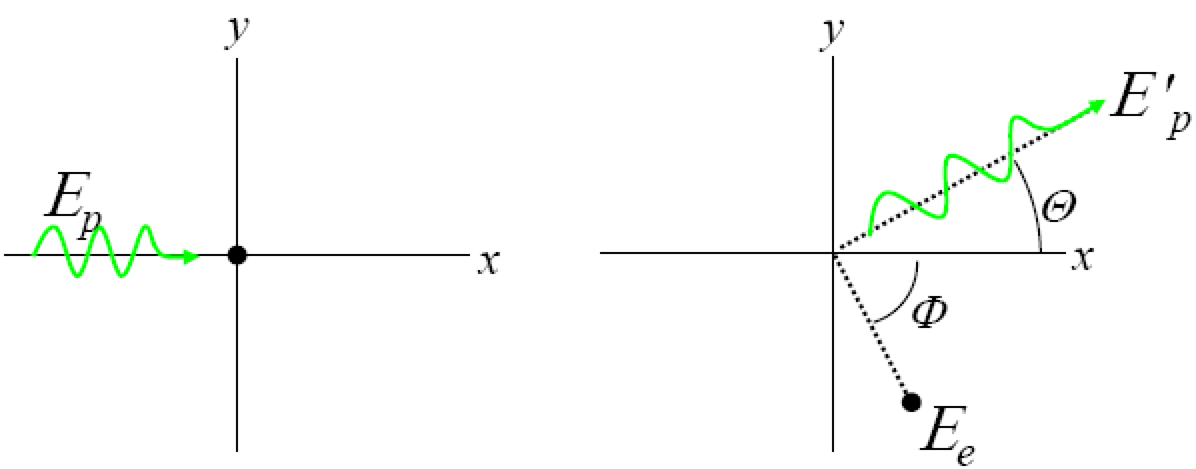






Compton Scattering

The x-ray photon interacts with one of the weakly bound electrons of the atom. This electron can be considered free because $_{Ex\text{-}ray}$*≈1-100 keV>>* $E_{e,b}$ *≈ few eV.*

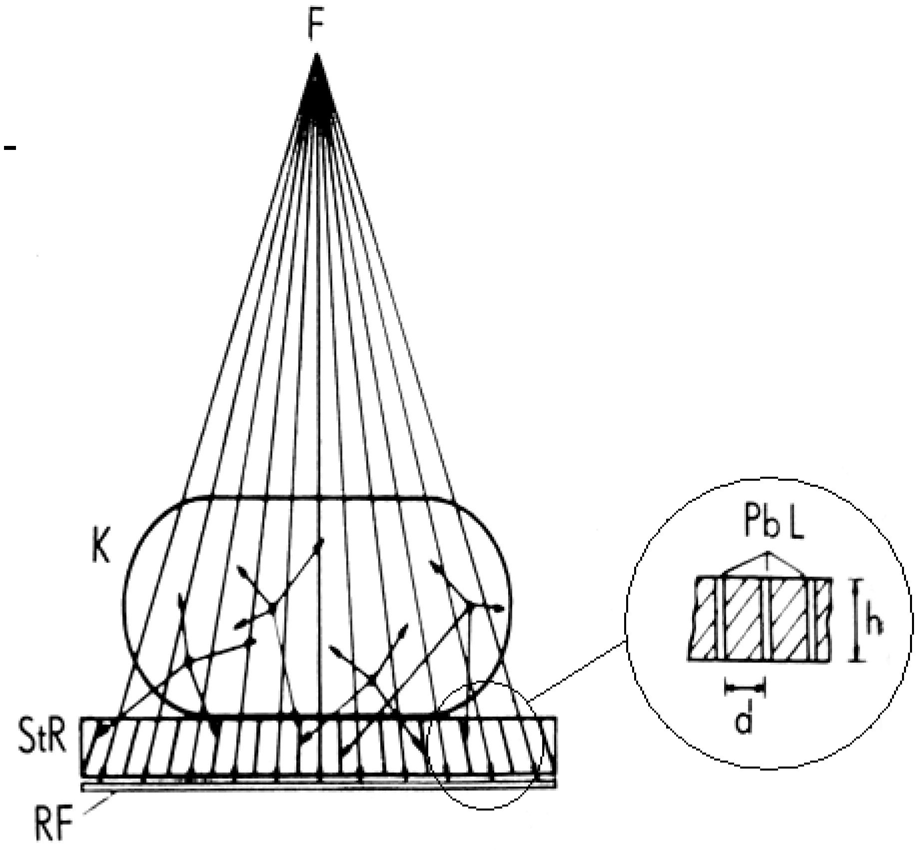


Inelastic scattering process:

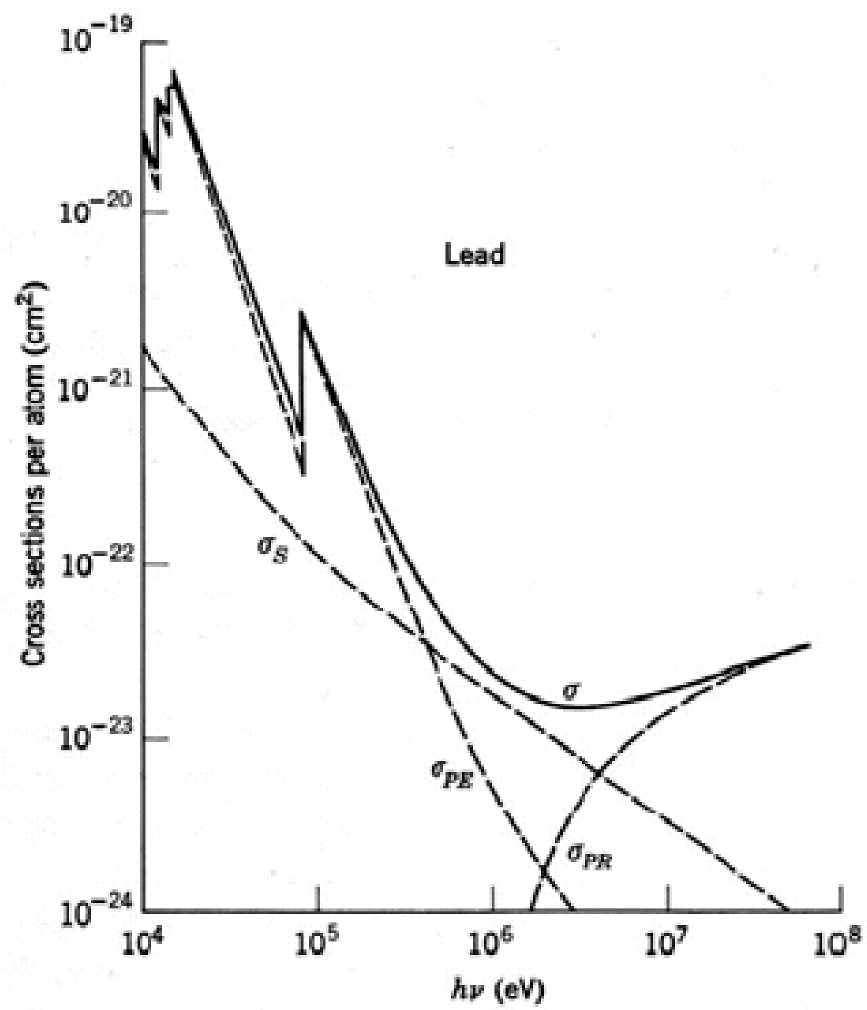
$$E'_p = E_p - E_e = \frac{E_p}{1+\alpha(1-\cos\theta)}; \qquad \alpha = \frac{E_p}{m_e c^2} = \frac{E_p}{511\,\mathbf{keV}}$$

Collimator

•Scattered x-rays are partially eliminated by secondary filter - collimator



Cross-section Examples



X-Ray Dosimetry I

X-ray exposure *X is quantified by measuring the number of free charge carriers (positive ions + electrons) generated in air at standard conditions. This leads to the traditional unit for exposure, the Roentgen [R]*

$$1\ \text{R} = 2.58 \times 10^{-4}\ \text{C/kg}$$

The *radiation absorbed dose D [Rad] is defined as*

$$1\ \text{Rad} = 100\ \text{erg/g} = 0.001\ \text{J/kg} = 0.001\ \text{Gray [Gy]}$$

Energy it takes to produce one ion pair in air: 33.97 eV(= 33.97 J/C)
⇒energy absorbed in air for 1 R:

$$2.58 \times 10^{-4}\ \text{C/kg} \times 33.97\ \text{J/C} = 0.00876\ \text{J/kg} = 87.6\ \text{ergs/g} = 0.876\ \text{Rad}$$

Conversion from exposure to dose:

$$D_{\text{air}}\ [\text{Rad}] = 0.876 \times X\ [\text{R}]$$

SI units

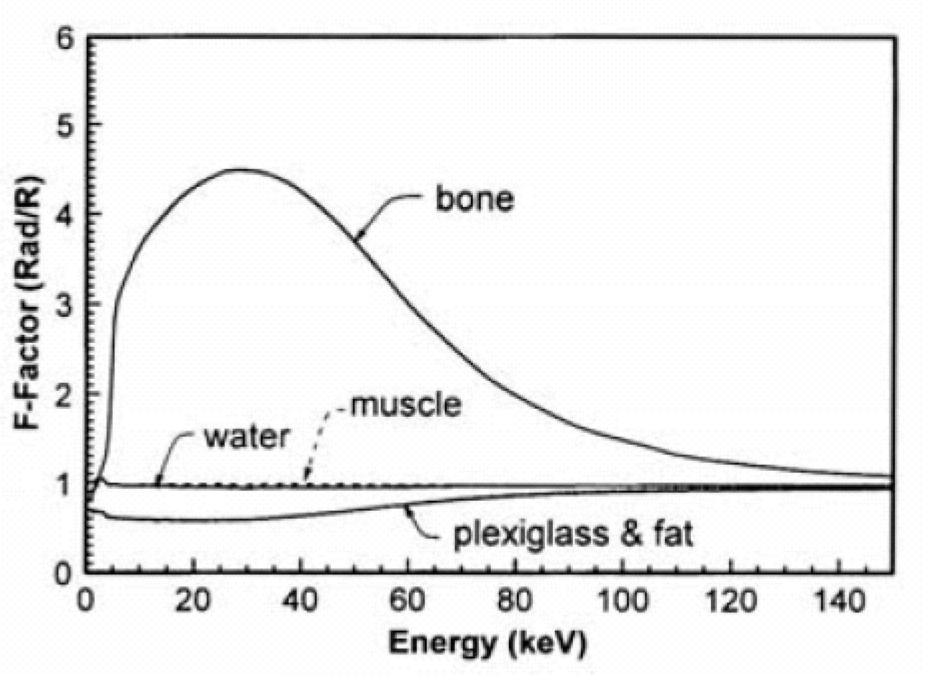
X-Ray Dosimetry II

Absorbed dose depends on material; for medium other than air:

$$D_{\text{med}} = D_{\text{air}} \frac{(\mu/\rho)_{\text{med}}}{(\mu/\rho)_{\text{air}}} = \underbrace{0.876 \frac{(\mu/\rho)_{\text{med}}}{(\mu/\rho)_{\text{air}}}}_{\text{F factor, } f} \times X \quad ^{*)}$$

Energy dependent F factor (Roentgen-to-Radconversion factor):

$$D_{\text{med}} = f X$$



*) to be precise, μ should be replaced by μ_{en}, the mass-energy absorption coefficient, which is a measure for the actually deposited radiation energy. For low Z: $\mu_{\text{en}} \approx \mu$

Biological effects of Ionizing Radiation

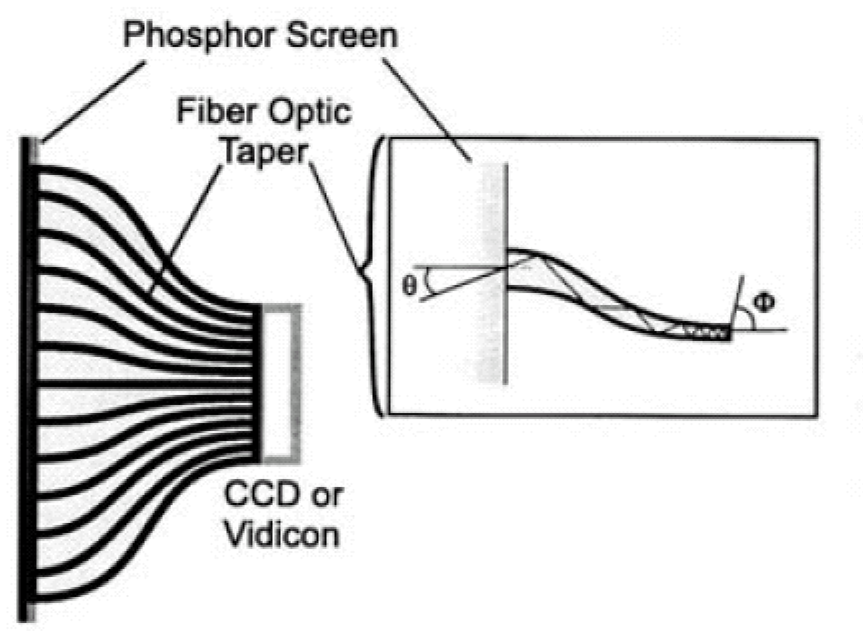
- Damage depends on deposited (= absorbed) energy (intensity $\times$ time) per tissue volume
- Threshold: No minimum level is known, below which damage occurs
- Exposure time: Because of recovery, a given dose is less harmful if divided
- Exposed area: The larger the exposed area the greater the damage (collimators, shields!)
- Variation in Species / Individuals: LD 50/30 (lethal for 50% of a population over 30 days, humans ~450 rads/ whole body irradiation)
- Variation in cell sensitivity: Most sensitive are nonspecialized, rapidly dividing cells (Most sensitive: White blood cells, red blood cells, epithelial cells. Less sensitive: Muscle, nerve cells)
- Short/long term effects: Short term effects for unusually large (> 100 rad) doses (nausea, vomiting, fever, shock, death); long term effects (carcinogenic/genetic effects) even for diagnostic levels $\Rightarrow$ maximum allowable dose 5 R/yr and 0.2 R/working day [Nat. Counc. on Rad. Prot. and Meas.]

X-ray Doses for Various Procedures

X-ray procedure	Exposure [mR]
Chest	20
Brain	250
Abdomen	550
Dental	650
Breast	54
Xeromammography	200
CT/slice	1000

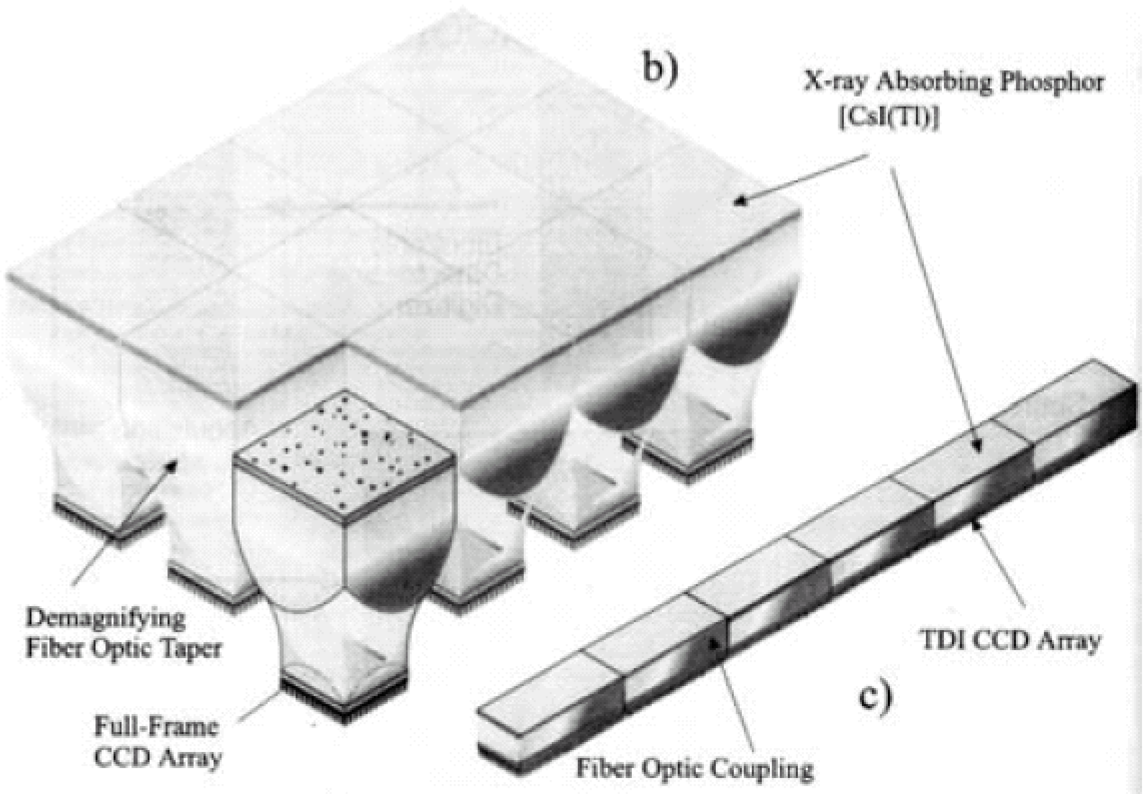
Small CCD Based Detectors

Small format (5 × 5 cm) for guidance of stereotacticbreast biopsy procedures. Phosphor is optically coupled by lens or fiber taper to 1k × 1k CCD array (real-time imaging).



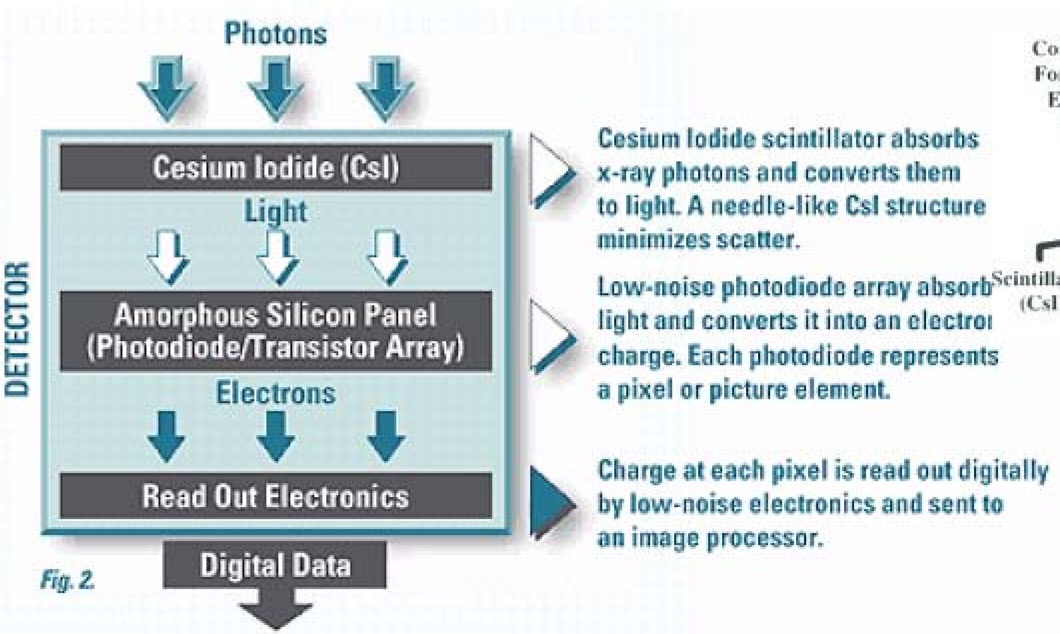
Large Sized CCD Based Detectors

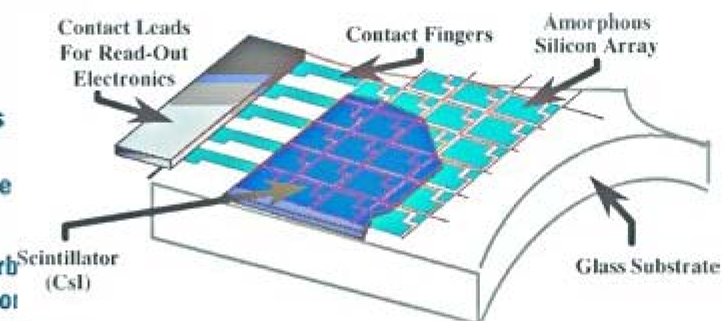
- CsI layer coupled by demagnifyingfiber tapers to CCD. Mosaic of 3 × 4 such units to cover necessary area.
- Slot configuration is scanned during acquisition process. Excellent scatter rejection, slower, scanning X-ray source required.



Flat Panels Detectors

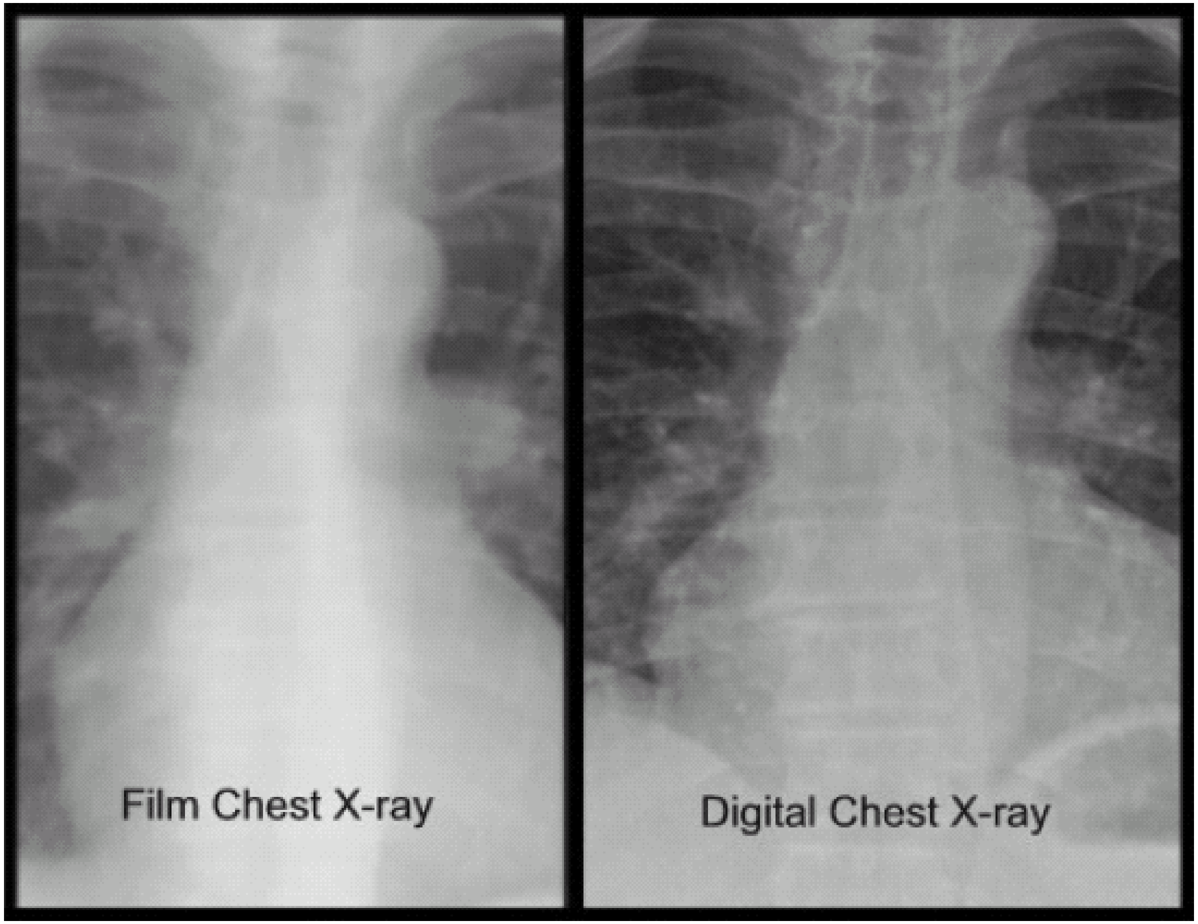
• CsI layer deposited directly on array of Si photodiodes with TFT switching matrix (GE 2000, first FDA approved full-digital system [11 yrs, 130 million])





• Direct conversion of x-ray into charge using higher-*Z semiconductors (lead iodide, selenium, zinc cadmium telluride, thallium bromide)*

Comparison Analog-Digital



X-Rays Devices